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# Adaptive Hybrid Position/Force Control of Robotic Manipulators

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## 1. Abstract

In this paper, the problem of position and force control for the compliant motion of the manipulators is considered. The external force and the position of the end-effector are related by a second order impedance function. The force control problem is then translated into a position control problem. For that, an adaptive controller is designed to achieve the compliant motion. The design uses the Liapunov's direct method to derive the adaptation law. The stability of the process is guaranteed from the Liapunov's stability theory. The controller does not require the knowledge of the system parameters for the implementation, and hence is easy for applications.

## 2. Introduction

While position control is appropriate when a manipulator is following a trajectory through space, when any contact is made between the end-effector and the manipulator's environment, position control may not suffice. Precise control of manipulators, in the face of uncertainties and variations in their environments, is a prerequisite to feasible application of robot manipulators to complex handling and assembly problems, in industry and space. An important step toward achieving such control may be taken by providing manipulator hands with sensors that provide information about the progress of interactions with the environment. Properly applied force control can reduce the positioning accuracy necessary to perform a given task accurately, and in fact make possible assembly tasks which would be otherwise impossible.

The problem of position/force control has attracted many researchers in the recent past years [1-5]. Among these works one can distinguish two different approaches. The first approach is aimed at providing the user with a means of specifying and controlling forces and positions in a non-conflicting way, [1-3]. This involves specification of a set of position controlled axes and an orthogonal set of force controlled axes. The second approach is aimed at developing a relationship between interaction forces and manipulator positions, [4,5]. This way, by controlling the manipulator position and specifying its relationship to the interaction forces, a designer can ensure that the manipulator will be able to maneuver in a constrained environment while maintaining appropriate contact forces.

In the first group, Paul and Shimano [1] partition the cartesian space and find the best joints to force servo to approximate the desired force and position commands. Raibert and Craig [2] involve all joints in satisfying the cartesian position and force commands simultaneously. Whitney [3] arrives at a single loop velocity control scheme with the net effect of controlling the contact force. In that paper, the impedance matrix approach establishes a connection between the two different approaches mentioned above. In all the above works, the structure of the controller depends on the kinematics and dynamics of the manipulator and of the environment. That is, if the end-effector of a manipulator in motion encounters a point with new constraint, then the controller structure must be changed. In the second group, Salisbury [4] defines a linear static function that relates interaction forces to end-effector position, by a stiffness matrix in a cartesian coordinate frame. Monitoring this relationship ensures that the manipulator will be able to maneuver successfully in a constrained environment. Kazerooni, et. al. [5] extend the previous work [4] and define a generalized mechanical impedance for the manipulator which is used for the compliant motion control. Their approach is an extended frequency domain approach of Salisbury's stiffness control. Also, their design is stable and shows robustness in the face of bounded uncertainties. In the second group approach, the controller's structure does not depend on the kinematics and dynamics of the manipulator and that of the environment. However, in both groups, the controller requires the knowledge of the parameters of the system.

In this work, the concept of mechanical impedance, [4,5] is used in order to relate the external forces to the position and orientation of the end-effector. Hence, the problem of force control is recasted in the position control problem. The objective is to design a controller for the manipulator, so that the perturbed dynamic relationship for the overall system is given by a second order impedance function. For that, a model reference adaptive controller is designed [6,7], where the desired impedance function is used to select the adaptive control model. The direct method of Liapunov is used for the derivation of adaptation laws. This guarantees the stability of the overall system.

### 3. Manipulator Dynamics

Consider a manipulator with  $n$  joints, providing  $n$  degrees of freedom. The dynamic equation of such manipulator is given by

$$M(q) \ddot{q} + h(q, \dot{q}) + g(q) = T \quad (1)$$

where  $q$  is the  $n$ -dimensional vector of joint angular positions,  $\dot{q}$  and  $\ddot{q}$  are, respectively, the vectors of joint angular velocities and joint angular accelerations,  $M(q)$  is the  $n \times n$  symmetric, positive definite inertia matrix of the manipulator,  $h(q, \dot{q})$  is the  $n$ -dimensional vector of Coriolis and centrifugal forces,  $g(q)$  is the  $n$ -dimensional vector of gravitational forces, and  $T$  is the  $n$ -dimensional vector of torque inputs, applied to the manipulator.

Let  $\delta q$  be the perturbation of the joint angular position vector  $q$ , from  $q_0$ , and  $\delta T$  be the perturbation of the input torques, from  $T_0$ . Then the linearized dynamic equation is given by

$$M(q_0) \delta \ddot{q} + G(q_0) \delta q = \delta T \quad (2)$$

where,  $G(q_0) = [\partial g / \partial q_1, \dots, \partial g / \partial q_n]$  for  $q = q_0$ .

The joint input torques applied to the manipulator, the external forces on the end-effector, and the actuator torques are related by

$$\delta T = L \delta T_a + J_c^T \delta F \quad (3)$$

where  $\delta T$ ,  $\delta T_a$  and  $\delta F$  are  $n$ -dimensional perturbations of the joint input torques, the actuator torques and the end-effector external forces, and  $J_c$  is the Jacobian matrix which transform joint angle coordinates to end-effector position and orientation. Also, the dynamic equation of actuators are approximately given by

$$\delta \dot{T}_a = A_a \delta T_a + B_a \delta U \quad (4)$$

where

$$A_a = \text{diag} [-\lambda_{a1}, \dots, -\lambda_{an}]$$

$$B_a = \text{diag} [b_1, \dots, b_n]$$

and  $\delta U$  is the  $n$ -dimensional vector of actuator inputs, [5].

From equations (2), (3), and (4), the dynamic equation of the manipulator and the actuator is given by

$$\begin{aligned} \delta \dot{X} &= A \delta X + B \delta U + D \delta F \\ \delta q &= C \delta X \end{aligned} \quad (5)$$

where

$$\delta X = [\delta q^T, \delta \dot{q}^T, \delta T_a^T]^T \in R^{3n}$$

$$A = \begin{bmatrix} 0 & I & 0 \\ -M^{-1}G & 0 & M^{-1}L \\ 0 & 0 & A_a \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ B_a \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ M^{-1}J_c^T \\ 0 \end{bmatrix}$$

$$C = [I \quad 0 \quad 0]$$

and the pairs  $(A, B)$  and  $(A, C)$  are respectively controllable and observable.

#### 4. Model Reference Adaptive Control

In this section, a controller is designed so that the dynamic perturbation equation of the overall closed-loop manipulator system is given according to the inverse of the desired impedance. To achieve this, a model reference adaptive control strategy is employed. The reference model is chosen such that its transfer matrix is identical to the inverse of the desired mechanical impedance. Hence, the dynamic equation of the reference model is given by

$$\begin{aligned}\dot{\delta X}_m &= A_m \delta X_m + B_m \delta F \\ \delta q_m &= C_m \delta X_m\end{aligned}\quad (6)$$

where

$$A_m = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ A_{m0} & A_{m1} & A_{m2} \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ 0 \\ B_{m0} \end{bmatrix}$$

$$C_m = [I \quad 0 \quad 0]$$

such that  $\delta X_m$  is the  $3n$ -dimensional incremental vector of model's joints and actuators values,  $\delta F$  is the  $n$ -dimensional vector of incremental external forces. The transfer function matrix of the model is given by

$$G_m(s) = \delta q_m(s)/\delta F(s) = C_m[sI - A_m]^{-1} B_m$$

such that the two dominant poles of the model are given by

$$J_c G_m(s) = (Js^2 + k_1 s + k_0)^{-1} \quad (7)$$

where  $J_c$  is the Jacobian matrix, and  $J$ ,  $k_0$ , and  $k_1$  are respectively the inertia matrix, stiffness matrix, and damping matrix of the desired mechanical impedance, given by

$$\delta F/\delta Y_m = (Js^2 + k_1 s + k_0), \quad \delta Y_m = \text{model's spatial displacement.}$$

Let us define the state error to be

$$e = \delta X_m - \delta X. \quad (8)$$

Subtracting equation (6) from (5), we get the dynamic equation of the state error as

$$\dot{e} = A_m e + (A_m - A)\delta X + (B_m - D)\delta F - B\delta U. \quad (9)$$

Let us now choose the input torque to be

$$\delta U = K_x \delta X + K_F \delta F + K_e e \quad (10)$$

where  $K_x$ ,  $K_F$ ,  $K_e$  are variable gain matrices with appropriate dimensions. Plugging  $\delta U$  from (10) into (9), we get

$$\dot{e} = (A_m - BK_e)e + (A_m - A - BK_x)\delta X + (B_m - D - BK_F)\delta F. \quad (11)$$

The problem, now, is how to vary the feedback and the feedforward gain matrices,  $K_x$ ,  $K_F$  and  $K_e$ , such that equation (11) is stable and the state error  $e$  approaches zero, according to a prespecified transient behavior.

To achieve perfect model following, the state error and its derivative should become zero, that is  $e = \dot{e} = 0$ . The conditions for perfect model following are given by

$$\begin{aligned}B_m - D - BK_F &= 0 \\ A_m - A - BK_x &= 0 \\ \bar{A} - A_m + BK_e &= 0\end{aligned}\quad (12)$$

Furthermore, under perfect model following conditions in (12) the error equation (11) will become

$$\dot{e} = \bar{A} e. \quad (13)$$

that is, the transient behavior of the state error is determined by the constant matrix  $\bar{A}$ , which is defined by the designer and is Hurwitz.

## 5. Adaptation Law

The controller gains of the adaptive system should be adjusted such that the overall closed-loop system is stable and follows the reference model. The direct method of Liapunov may be chosen for determining the adaptation law, [6,7].

Let the corresponding Liapunov function for adaptation be given by

$$V = ||e||_P + ||B_m - D - BK_F||_R + ||A_m - A - BK_X||_S + ||\bar{A} - A_m + BK_e||_M \quad (14)$$

where P, R, and S are  $3n \times 3n$  arbitrary positive definite symmetric constant matrices. Also, the quadratic norm for any matrix F and any positive definite symmetric constant matrix G is defined by

$$||F||_G = \text{tr}[F^T G F], \text{ where tr} = \text{trace}.$$

The function V is positive definite, except when there is a perfect model matching it becomes zero. Differentiating V, we get

$$\begin{aligned} \dot{V} = & e^T (P\bar{A} + \bar{A}^T P) e \\ & + 2\text{tr}[(B_m - D - BK_F)^T [Pe \delta F^T + R \frac{d}{dt} (B_m - D - BK_F)]] \\ & + 2\text{tr}[(A_m - A - BK_X)^T [Pe \delta X^T + S \frac{d}{dt} (A_m - A - BK_X)]] \\ & + 2\text{tr}[(\bar{A} - A_m + BK_e)^T [Pe e^T + M \frac{d}{dt} (\bar{A} - A_m + BK_e)]] \end{aligned} \quad (15)$$

Also notice that, since matrix  $\bar{A}$  is Hurwitz, then for any given positive definite symmetric matrix Q there exists a positive definite symmetric matrix P such that

$$P\bar{A} + \bar{A}^T P = -Q$$

Now, for the stability,  $\dot{V}$  should be negative. One way to satisfy this is to choose

$$\begin{aligned} \dot{K}_F &= B^\dagger R^{-1} Pe \delta F^T \\ \dot{K}_X &= B^\dagger S^{-1} Pe \delta X^T \\ \dot{K}_e &= -B^\dagger M^{-1} Pe e^T \end{aligned} \quad (16)$$

where  $B^\dagger = [0, 0, B^{-1}]$  is the pseudo-inverse of B. However, since R, S and M are arbitrary matrices, we can choose them such that  $RB^\dagger = \alpha I$ ,  $SB^\dagger = \beta I$  and  $MB^\dagger = \gamma I$ , where  $\alpha, \beta, \gamma$  are positive scalars. Then denoting  $E = [0, 0, I]$ , the adaptation laws can be given by

$$\begin{aligned} \dot{K}_F &= \alpha E Pe \delta F^T \\ \dot{K}_X &= \beta E Pe \delta X^T \\ \dot{K}_e &= -\gamma E Pe e^T \end{aligned} \quad (17)$$

With these adaptation laws, the derivative of the Liapunov function,  $\dot{V}$ , is given by

$$\dot{V} = -||e||_Q = -e^T Q e < 0$$

which is negative for non-zero state error, (i.e.  $e \neq 0$ ). This guarantees the asymptotic stability of the equilibrium point,  $e = 0$ .

The proposed design adaptively controls both the position and the end-effector force, and is appropriate for compliant motion of the robotic manipulators. The proposed adaptive controller is shown in Figure 1.

Moreover, if the spatial displacement and velocity can be directly measured, then the knowledge of  $J_c$  is not necessary for the implementation of the adaptive controller.

## 6. Conclusions

In this paper, the definition of mechanical impedance used in [4,5], is employed. The external force and the position of the end-effector are related by a second order impedance function. The force control problem is then translated to a position control problem. An adaptive controller is designed for the latter problem to achieve the compliant motion for the manipulator. The design uses the Liapunov's direct method to derive the adaptation law. The stability of the process is guaranteed from the Liapunov's stability theory. The major advantage of this method is that the controller does not depend on the knowledge of the system parameters and those of the environment. It uses the measured forces at the end-effector and the position and velocity of the end-effector in the joint space. The controller is simple and can be easily implemented by small computers.

## 7. References

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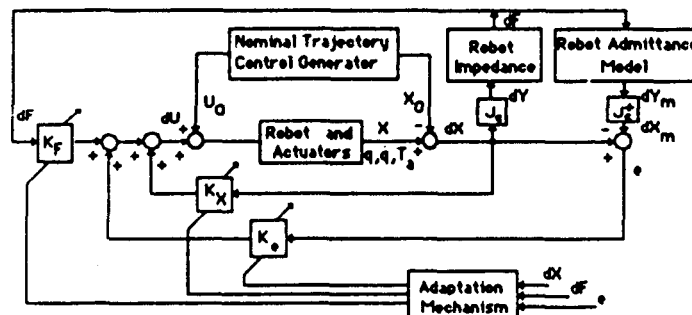


Figure 1. Adaptive Position/Force Controller for Robot